Reglas de reducción:

Primera forma

$$\left.\begin{matrix} RT(180° \pm \theta) \\ RT(360° - \theta) \end{matrix}\right\} = \pm RT(\theta)$$

$$\left.\begin{matrix} RT(\pi \pm \theta) \\ RT(2\pi - \theta) \end{matrix}\right\} = \pm RT(\theta)$$

también

$$RT(360°n + \theta) = RT(\theta);\ n \in \mathbb{Z}$$

$$RT(2\pi n + \theta) = RT(\theta);\ n \in \mathbb{Z}$$

Segunda forma

$$\left.\begin{matrix} RT(90° \pm \theta) \\ RT(270° \pm \theta) \end{matrix}\right\} = \pm CoRT(\theta)$$

$$\left.\begin{matrix} RT\left(\frac{\pi}{2} \pm \theta\right) \\ RT\left(\frac{3\pi}{2} \pm \theta\right) \end{matrix}\right\} = \pm CoRT(\theta)$$

Si $\theta + \alpha = \pi$ $\Rightarrow$

$\sen(\theta) = \sen(\alpha)$ | $\tan(\theta) = -\tan(\alpha)$ | $\sec(\theta) = -\sec(\alpha)$

$\cos(\theta) = -\cos(\alpha)$ | $\cot(\theta) = -\cot(\alpha)$ | $\csc(\theta) = \csc(\alpha)$

Si $\theta - \alpha = \pi$ $\Rightarrow$

$\sen(\theta) = -\sen(\alpha)$ | $\tan(\theta) = \tan(\alpha)$ | $\sec(\theta) = -\sec(\alpha)$

$\cos(\theta) = -\cos(\alpha)$ | $\cot(\theta) = \cot(\alpha)$ | $\csc(\theta) = -\csc(\alpha)$

Si $\theta + \alpha = 2\pi$ $\Rightarrow$

$\sen(\theta) = -\sen(\alpha)$ | $\tan(\theta) = -\tan(\alpha)$ | $\sec(\theta) = \sec(\alpha)$

$\cos(\theta) = \cos(\alpha)$ | $\cot(\theta) = -\cot(\alpha)$ | $\csc(\theta) = -\csc(\alpha)$

Para cambiar de signo al ángulo

$\sen(-\theta) = -\sen(\theta)$ | $\tan(-\theta) = -\tan(\theta)$ | $\cos(-\theta) = \cos(\theta)$

$\csc(-\theta) = -\csc(\theta)$ | $\cot(-\theta) = -\cot(\theta)$ | $\sec(-\theta) = \sec(\theta)$

Forma Generalizada

$$RT(180°k \pm \theta) = \pm RT(\theta)$$

$$RT\big((2k+1)90° \pm \theta\big) = \pm CoRT(\theta)$$

$$RT(\pi k \pm \theta) = \pm RT(\theta)$$

$$RT\left((2k+1)\frac{\pi}{2} \pm \theta\right) = \pm CoRT(\theta)$$

Identidades Trigonométricas para un mismo Arco

Identidades Recíprocas

$$\operatorname{sen}(x)\cdot\csc(x)=1$$

$$\cos(x)\cdot\sec(x)=1$$

$$\tan(x)\cdot\cot(x)=1$$

Identidades Por División

$$\tan(x)=\frac{\operatorname{sen}(x)}{\cos(x)}$$

$$\cot(x)=\frac{\cos(x)}{\operatorname{sen}(x)}$$

Identidades Pitagóricas

$$\operatorname{sen}^2(x)+\cos^2(x)=1$$

$$\sec^2(x)=1+\tan^2(x)$$

$$\csc^2(x)=1+\cot^2(x)$$

Identidades Auxiliares

1. $\operatorname{sen}^4(x)+\cos^4(x)=1-\mathbf{2}\operatorname{sen}^2(x).\cos^2(x)$
2. $\operatorname{sen}^6(x)+\cos^6(x)=1-\mathbf{3}\operatorname{sen}^2(x).\cos^2(x)$
3. $\tan(x)+\cot(x)=\sec(x).\csc(x)$
4. $\sec^2(x)+\csc^2(x)=\sec^2(x).\csc^2(x)$
5. $(\operatorname{sen}(x)\pm\cos(x))^2=1\pm 2\operatorname{sen}(x)\cos(x))$
6. $(\operatorname{sen}(x)+\cos(x)+1)(\operatorname{sen}(x)+\cos(x)-1)=2\operatorname{sen}(x)\cos(x)$
7. $(1\pm\operatorname{sen}(x)\pm\cos(x))^2=2(1\pm\operatorname{sen}(x))(1\pm\cos(x))$
8. $\sec(x)+\tan(x)=\dfrac{1}{\sec(x)-\tan(x)}$
9. $\csc(x)+\cot(x)=\dfrac{1}{\csc(x)-\cot(x)}$

Identidades Auxiliares Adicionales

1. $\sec^4(x) + \tan^4(x) = 1 + 2\sec^2(x).\tan^2(x)$
2. $\sec^6(x) - \tan^6(x) = 1 + 3\sec^2(x).\tan^2(x)$
3. $\csc^4(x) + \cot^4(x) = 1 + 2\csc^2(x).\cot^2(x)$
4. $\csc^6(x) - \cot^6(x) = 1 + 3\csc^2(x).\cot^2(x)$
5. $\text{sen}^2(x) + \cos^4(x) = 1 - \text{sen}^2(x).\cos^2(x)$
6. $\cos^2(x) + \text{sen}^4(x) = 1 - \text{sen}^2(x).\cos^2(x)$
7. $\tan^2(x) - \text{sen}^2(x) = \tan^2(x).\text{sen}^2(x)$
8. $\cot^2(x) - \cos^2(x) = \cot^2(x).\cos^2(x)$
9. Si $a\,\text{sen}(x) + b\cos(x) = \sqrt{a^2 + b^2}$

   $\Longrightarrow \text{sen}(x) = \frac{a}{\sqrt{a^2+b^2}}$ y $\cos(x) = \frac{b}{\sqrt{a^2+b^2}}$
10. $\forall\, a, b, x \in \mathbb{R}$:

    $-\sqrt{a^2 + b^2} \leq a\,\text{sen}(x) + b\cos(x) \leq \sqrt{a^2 + b^2}$
11. $\dfrac{\cos(x)}{1 \pm \text{sen}(x)} = \dfrac{1 \mp \text{sen}(x)}{\cos(x)}$
12. $\dfrac{\text{sen}(x)}{1 \pm \cos(x)} = \dfrac{1 \mp \cos(x)}{\text{sen}(x)}$

Expresiones Notables de Algebra

$(a \pm b)^2 = a^2 + b^2 \pm 2ab$

$a^2 - b^2 = (a + b)\,(a - b)$

$(a + b)^2 - (a - b)^2 = 4ab$

$ab = 1 \rightarrow (a \pm b)^2 = a^2 + b^2 \pm 2$

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$ab = 1 \rightarrow (a + b)^3 = a^3 + b^3 + 3(a + b)$

$a + b + c = 0 \rightarrow a^3 + b^3 + c^3 = 3abc$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Identidades Trigonométricas para la Suma y Diferencia de dos Arcos

Identidades Básicas

$$\mathrm{sen}(x+y) = \mathrm{sen}(x)\cdot\cos(y) + \cos(x)\cdot\mathrm{sen}(y)$$

$$\mathrm{sen}(x-y) = \mathrm{sen}(x)\cdot\cos(y) - \cos(x)\cdot\mathrm{sen}(y)$$

$$\cos(x-y) = \cos(x)\cdot\cos(y) + \mathrm{sen}(x)\cdot\mathrm{sen}(y)$$

$$\cos(x+y) = \cos(x)\cdot\cos(y) - \mathrm{sen}(x)\cdot\mathrm{sen}(y)$$

$$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\cdot\tan(y)}$$

$$\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\cdot\tan(y)}$$

$$\cot(x+y) = \frac{\cot(x)\cot(y)-1}{\cot(y)+\cot(x)}$$

$$\cot(x-y) = \frac{\cot(x)\cot(y)+1}{\cot(y)-\cot(x)}$$

Identidades Auxiliares

$$\tan(\alpha)+\tan(\beta)+\tan(\alpha)\cdot\tan(\beta)\cdot\tan(\alpha+\beta) = \tan(\alpha+\beta)$$

$$\tan(\alpha)-\tan(\beta)-\tan(\alpha)\cdot\tan(\beta)\cdot\tan(\alpha-\beta) = \tan(\alpha-\beta)$$

$$\mathrm{sen}(\alpha+\beta)\cdot\mathrm{sen}(\alpha-\beta) = \mathrm{sen}^2(\alpha)-\mathrm{sen}^2(\beta)$$

$$\cos(\alpha+\beta)\cdot\cos(\alpha-\beta) = \cos^2(\alpha)-\mathrm{sen}^2(\beta)$$

$$\frac{\mathrm{sen}(\alpha\pm\beta)}{\cos(\alpha)\cdot\cos(\beta)} = \tan(\alpha)\pm\tan(\beta)$$

$$\frac{\mathrm{sen}(\alpha\pm\beta)}{\mathrm{sen}(\alpha)\cdot\mathrm{sen}(\beta)} = \cot(\beta)\pm\cot(\alpha)$$

$$\frac{\cos(\alpha\pm\beta)}{\cos(\alpha)\cos(\beta)} = 1\mp\tan(\alpha)\tan(\beta)$$

$$\frac{\cos(\alpha\pm\beta)}{\mathrm{sen}(\alpha)\,\mathrm{sen}(\beta)} = \cot(\alpha)\cot(\beta)\mp 1$$

Si a y b son constantes reales, con $a>0$ $b>0$

$$a\cdot\mathrm{sen}(x) \pm b\cdot\cos(x) = \sqrt{a^2+b^2}\,\mathrm{sen}(x\pm\theta)$$

donde: $\tan(\theta) = \dfrac{b}{a}$ con $0<\theta<{}^{\pi}/_{2}$

Identidades Trigonométricas Condicionales con Tres Arcos

$$\boldsymbol{\alpha + \beta + \theta = (2k+1)\frac{\pi}{2}\ ;\ k \in \mathbb{Z}}$$

$$\boxed{\tan(\alpha)\cdot\tan(\beta) + \tan(\beta)\cdot\tan(\theta) + \tan(\theta)\cdot\tan(\alpha) = 1}$$

$$\boxed{\cot(\alpha) + \cot(\beta) + \cot(\theta) = \cot(\alpha)\cdot\cot(\beta)\cdot\cot(\theta)}$$

$$\boldsymbol{\alpha + \beta + \theta = k\pi\ ;\ k \in \mathbb{Z}}$$

$$\boxed{\cot(\alpha)\cdot\cot(\beta) + \cot(\beta)\cdot\cot(\theta) + \cot(\theta)\cdot\cot(\alpha) = 1}$$

$$\boxed{\tan(\alpha) + \tan(\beta) + \tan(\theta) = \tan(\alpha)\cdot\tan(\beta)\cdot\tan(\theta)}$$

$$\boldsymbol{x + y + z = 0}$$

$$\boxed{\cos^2(x) + \cos^2(y) + \cos^2(z) - 2\cos(x)\cos(y)\cos(z) = 1}$$

$$\boldsymbol{x + y + z = \frac{\pi}{2}}$$

$$\boxed{\operatorname{sen}^2(x) + \operatorname{sen}^2(y) + \operatorname{sen}^2(z) + 2\operatorname{sen}(x)\operatorname{sen}(y)\operatorname{sen}(z) = 1}$$

$$\boldsymbol{x + y + z = \pi}$$

$$\boxed{\cos^2(x) + \cos^2(y) + \cos^2(z) + 2\cos(x)\cos(y)\cos(z) = 1}$$

Identidades Trigonométricas para la Suma de Tres Arcos

$$\operatorname{sen}(x+y+z)=\operatorname{sen}(x)\cos(y)\cos(z)+\operatorname{sen}(y)\cos(x)\cos(z)+\operatorname{sen}(z)\cos(x)\cos(y)-\operatorname{sen}(x)\operatorname{sen}(y)\operatorname{sen}(z)$$

$$\cos(x+y+z)=\cos(x)\cos(y)\cos(z)-\operatorname{sen}(x)\operatorname{sen}(y)\cos(z)-\operatorname{sen}(x)\operatorname{sen}(z)\cos(y)-\operatorname{sen}(y)\operatorname{sen}(z)\cos(x)$$

$$\tan(x+y+z)=\frac{\tan(x)+\tan(y)+\tan(z)-\tan(x)\tan(y)\tan(z)}{1-\tan(x)\tan(y)-\tan(y)\tan(z)-\tan(x)\tan(z)}$$

$$\frac{\operatorname{sen}(x+y+z)}{\cos(x)\cos(y)\cos(z)}=\tan(x)+\tan(y)+\tan(z)-\tan(x)\tan(y)\tan(z)$$

$$\frac{\operatorname{sen}(x+y+z)}{\operatorname{sen}(x)\operatorname{sen}(y)\operatorname{sen}(z)}=\cot(x)\cot(y)+\cot(y)\cot(z)+\cot(z)\cot(x)-1$$

$$\frac{\cos(x+y+z)}{\cos(x)\cos(y)\cos(z)}=1-\tan(x)\tan(y)-\tan(y)\tan(z)-\tan(x)\tan(z)$$

$$\frac{\cos(x+y+z)}{\operatorname{sen}(x)\operatorname{sen}(y)\operatorname{sen}(z)}=\cot(x)\cot(y)\cot(z)-\cot(x)-\cot(y)-\cot(z)$$

Identidades Trigonométricas para el Arco Doble

Identidades fundamentales

$$\mathrm{sen}(2\theta) = 2\mathrm{sen}(\theta)\cos(\theta)$$

$$\cos(2\theta) = \cos^2(\theta) - \mathrm{sen}^2(\theta)$$

$$\cos(2\theta) = 2\cos^2(\theta) - 1$$

$$\boldsymbol{2\cos^2(\theta) = 1 + \cos(2\theta)}$$

$$\cos(2\theta) = 1 - 2\mathrm{sen}^2(\theta)$$

$$\boldsymbol{2\mathrm{sen}^2(\theta) = 1 - \cos(2\theta)}$$

Identidades Auxiliares

$$\tan(\theta) + \cot(\theta) = 2\csc(2\theta)$$

$$\cot(\theta) - \tan(\theta) = 2\cot(2\theta)$$

$$\sec^2(\theta) + \csc^2(\theta) = 4\csc^2(2\theta)$$

$$\mathrm{sen}^4(x) + \cos^4(x) = \frac{3 + \cos(4x)}{4}$$

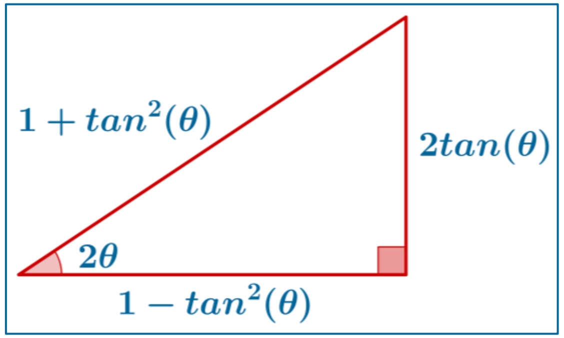
$$\mathrm{sen}^6(x) + \cos^6(x) = \frac{5 + 3\cos(4x)}{8}$$

$$8\mathrm{sen}^4(\theta) = 3 - 4\cos(2\theta) + \cos(4\theta)$$

$$8\cos^4(\theta) = 3 + 4\cos(2\theta) + \cos(4\theta)$$

$$(\mathrm{sen}(\theta) \pm \cos(\theta))^2 = 1 \pm \mathrm{sen}(2\theta)$$

$$\tan(2\theta) = \frac{2\tan(\theta)}{1 - \tan^2(\theta)}$$



$$\mathrm{sen}(2\theta) = \frac{2\tan(\theta)}{1 + \tan^2(\theta)}$$

$$\cos(2\theta) = \frac{1 - \tan^2(\theta)}{1 + \tan^2(\theta)}$$

Identidades Trigonométricas para el Arco Mitad

$$\operatorname{sen}\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos(\theta)}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{1+\cos(\theta)}}$$

Identidades Auxiliares

$$\cot\left(\frac{\theta}{2}\right) = \csc(\theta) + \cot(\theta)$$

$$\tan\left(\frac{\theta}{2}\right) = \csc(\theta) - \cot(\theta)$$

$$\tan(2\theta)\tan(\theta) = \sec(2\theta) - 1$$

$$\frac{\tan(2\theta)}{\tan(\theta)} = \sec(2\theta) + 1$$

$$\csc(\theta) + \csc(2\theta) + \csc(4\theta) + \csc(8\theta) + \cdots \csc(2^n\theta) = \cot\left(\frac{\theta}{2}\right) - \cot(2^n\theta); \quad \forall\, n \in \mathbb{Z}^+$$

$$\sec(\theta) + \tan(\theta) = \cot\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$$

$$\sec(\theta) - \tan(\theta) = \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$$

Identidades Trigonométricas para el Arco Triple

Identidades fundamentales

$$\operatorname{sen}(3x) = 3\operatorname{sen}(x) - 4\operatorname{sen}^3(x)$$

$$\mathbf{4\operatorname{sen}^3(x) = 3\operatorname{sen}(x) - \operatorname{sen}(3x)}$$

$$\cos(3x) = 4\cos^3(x) - 3\cos(x)$$

$$\mathbf{4\cos^3(x) = 3\cos(x) + \cos(3x)}$$

$$\tan(3x) = \frac{3\tan(x) - \tan^3(x)}{1 - 3\tan^2(x)}$$

Identidades Auxiliares

$$\operatorname{sen}(3x) = \operatorname{sen}(x)[2\cos(2x) + 1]$$

$$\cos(3x) = \cos(x)[2\cos(2x) - 1]$$

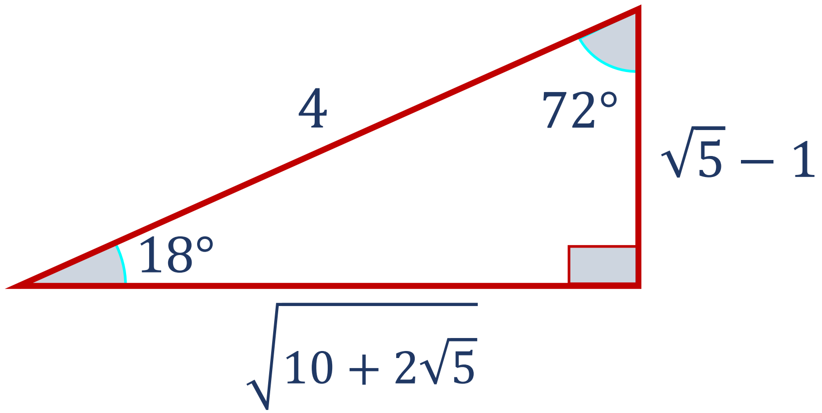
$$\tan(3x) = \tan(x)\left[\frac{2\cos(2x) + 1}{2\cos(2x) - 1}\right]$$

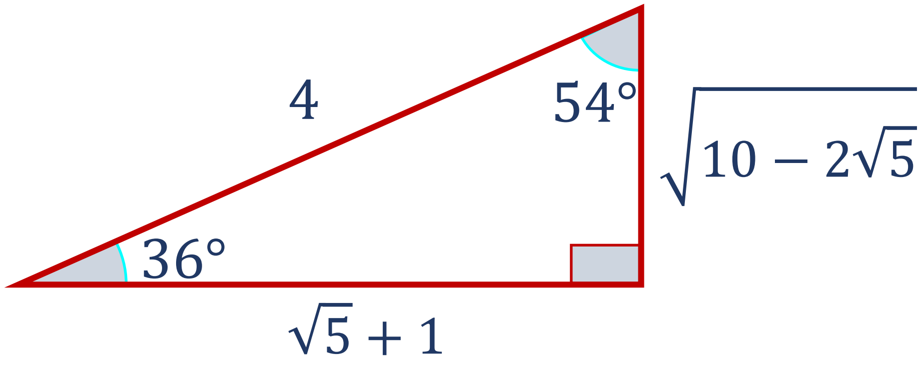
$$4\operatorname{sen}(x)\operatorname{sen}(60° - x)\operatorname{sen}(60° + x) = \operatorname{sen}(3x)$$

$$4\cos(x)\cos(60° - x)\cos(60° + x) = \cos(3x)$$

$$\tan(x)\tan(60° - x)\tan(60° + x) = \tan(3x)$$

Ángulos Notables





Transformaciones Trigonométricas

De Suma o Diferencia a Producto

$$\operatorname{sen}(\alpha) + \operatorname{sen}(\beta) = 2\operatorname{sen}\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\operatorname{sen}(\alpha) - \operatorname{sen}(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right)\operatorname{sen}\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos(\alpha) + \cos(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos(\alpha) - \cos(\beta) = -2\operatorname{sen}\left(\frac{\alpha+\beta}{2}\right)\operatorname{sen}\left(\frac{\alpha-\beta}{2}\right)$$

De Producto a Suma o Diferencia

$$2\operatorname{sen}(x)\cos(y) = \operatorname{sen}(x+y) + \operatorname{sen}(x-y)$$

$$2\cos(x)\operatorname{sen}(y) = \operatorname{sen}(x+y) - \operatorname{sen}(x-y)$$

$$2\cos(x)\cos(y) = \cos(x+y) + \cos(x-y)$$

$$2\operatorname{sen}(x)\operatorname{sen}(y) = \cos(x-y) - \cos(x+y)$$

Condicional: Si $\mathbf{A + B + C = 180^\circ}$, entonces:

$$\operatorname{sen}(A) + \operatorname{sen}(B) + \operatorname{sen}(C) = 4\cos\left(\frac{A}{2}\right)\cos\left(\frac{B}{2}\right)\cos\left(\frac{C}{2}\right)$$

$$\cos(A) + \cos(B) + \cos(C) - 1 = 4\operatorname{sen}\left(\frac{A}{2}\right)\operatorname{sen}\left(\frac{B}{2}\right)\operatorname{sen}\left(\frac{C}{2}\right)$$

$$\operatorname{sen}(2A) + \operatorname{sen}(2B) + \operatorname{sen}(2C) = 4\operatorname{sen}(A)\operatorname{sen}(B)\operatorname{sen}(C)$$

$$\cos(2A) + \cos(2B) + \cos(2C) + 1 = -4\cos(A)\cos(B)\cos(C)$$

$$\operatorname{sen}(x)+\operatorname{sen}(y)+\operatorname{sen}(z)-\operatorname{sen}(x+y+z)=4\operatorname{sen}\left(\frac{x+y}{2}\right)\operatorname{sen}\left(\frac{y+z}{2}\right)\operatorname{sen}\left(\frac{z+x}{2}\right)$$

$$\cos(x)+\cos(y)+\cos(z)+\cos(x+y+z)=4\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{y+z}{2}\right)\cos\left(\frac{z+x}{2}\right)$$

Suma de senos o cosenos de ángulos cuyas medidas están en progresión aritmética

$$\operatorname{sen}(x)+\operatorname{sen}(x+r)+\operatorname{sen}(x+2r)+\operatorname{sen}(x+3r)+\cdots+\operatorname{sen}(x+(n-1)r)=\frac{\operatorname{sen}\left(\frac{nr}{2}\right)}{\operatorname{sen}\left(\frac{r}{2}\right)}\cdot\operatorname{sen}\left(\frac{P\sphericalangle+U\sphericalangle}{2}\right)$$

$$\cos(x)+\cos(x+r)+\cos(x+2r)+\cos(x+3r)+\cdots+\cos(x+(n-1)r)=\frac{\operatorname{sen}\left(\frac{nr}{2}\right)}{\operatorname{sen}\left(\frac{r}{2}\right)}\cdot\cos\left(\frac{P\sphericalangle+U\sphericalangle}{2}\right)$$

n: es el número de términos

r: es la razón de la progresión aritmética del ángulo

$P\sphericalangle$: es la medida del primer ángulo de la serie ($P\angle=x$)

$U\sphericalangle$: es la medida del último ángulo de la serie ($U\angle=x+(n-1)r$)

PROPIEDAD

$$\sum_{k=1}^{n} \cos\left(\left(\frac{2k}{2n+1}\right)\pi\right) = \cos\left(\frac{2\pi}{2n+1}\right) + \cos\left(\frac{4\pi}{2n+1}\right) + \cdots + \cos\left(\frac{2n\pi}{2n+1}\right) = -\frac{1}{2}$$

$$\sum_{k=1}^{n} \cos\left(\left(\frac{2k-1}{2n+1}\right)\pi\right) = \cos\left(\frac{\pi}{2n+1}\right) + \cos\left(\frac{3\pi}{2n+1}\right) + \cdots + \cos\left(\frac{(2n-1)\pi}{2n+1}\right) = \frac{1}{2}$$

PRODUCTOS DE R.T. DE ARCOS DE LA FORMA $\left(\frac{\mathbf{k\pi}}{\mathbf{2n+1}}\right)$; $\mathbf{n, k \in \mathbb{Z}}$ y $\mathbf{k = 1, 2, .., n}$

i. $$\prod_{k=1}^{n} \operatorname{sen}\left(\frac{k\pi}{2n+1}\right) = \operatorname{sen}\left(\frac{\pi}{2n+1}\right)\operatorname{sen}\left(\frac{2\pi}{2n+1}\right) \ldots\ldots \operatorname{sen}\left(\frac{n\pi}{2n+1}\right) = \frac{\sqrt{2n+1}}{2^n}$$

ii. $$\prod_{k=1}^{n} \cos\left(\frac{k\pi}{2n+1}\right) = \cos\left(\frac{\pi}{2n+1}\right)\cos\left(\frac{2\pi}{2n+1}\right) \ldots\ldots \cos\left(\frac{n\pi}{2n+1}\right) = \frac{1}{2^n}$$

iii. $$\prod_{k=1}^{n} \tan\left(\frac{k\pi}{2n+1}\right) = \tan\left(\frac{\pi}{2n+1}\right)\tan\left(\frac{2\pi}{2n+1}\right) \ldots\ldots \tan\left(\frac{n\pi}{2n+1}\right) = \sqrt{2n+1}$$

Variación o Extensión de Expresiones Típicas, $\forall\, \mathbf{x} \in \mathbb{R}$

$$-1 \leq \operatorname{sen}(x) \leq 1$$

$$0 \leq 1 \pm \operatorname{sen}(x) \leq 2$$

$$-1 \leq \cos(x) \leq 1$$

$$0 \leq 1 \pm \cos(x) \leq 2$$

$$-\sqrt{a^2 + b^2} \leq a \cdot \operatorname{sen}(x) + b \cdot \cos(x) \leq \sqrt{a^2 + b^2}$$

$$-\sqrt{2} \leq \operatorname{sen}(x) \pm \cos(x) \leq \sqrt{2}$$

$$\forall n \in \mathbb{Z},\ \text{con } n \text{ impar},\ n \geq 3: \quad -1 \leq \operatorname{sen}^n(x) \pm \cos^n(x) \leq 1$$

$$-\frac{1}{2} \leq \operatorname{sen}(x)\cos(x) \leq \frac{1}{2}$$

$$0 \leq \operatorname{sen}^2(x)\cos^2(x) \leq \frac{1}{4}$$

$$\forall n \in \mathbb{Z}^+: \quad \frac{1}{2^{n-1}} \leq \operatorname{sen}^{2n}(x) + \cos^{2n}(x) \leq 1$$

$$\frac{1}{2} \leq \operatorname{sen}^4(x) + \cos^4(x) \leq 1$$

$$\frac{1}{4} \leq \operatorname{sen}^6(x) + \cos^6(x) \leq 1$$